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Void probability in the FRITIOF model

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Abstract. The void probability in the QCD-inspired FRITIOF model is studied for both unrestricted and restricted phase space. Comparison to phenomenological clan model is presented. The observed enhancement of void probability can be accommodated with the clan model when the randomly produced clans decay with the geometric distribution instead of the logarithmic one. A simple parameterization is introduced to reproduce the shift of weighting factor with the size of restricted phase space. The predictions to the multiplicity distributions of $\bar{p}p$ collision at $\sqrt{s} = 10$ TeV is presented.

1 Introduction

The multiplicity distribution is the most basic measurement in the phenomena of multiple production in high energy collisions. The detailed shape of the distribution reflects the underlying dynamics and statistics. In this paper we study the multiplicity distribution in the low multiplicity region, especially the void probability.

The void probability is defined as the probability for events with zero particle. It is easy to understand the non-zero void probability for the multiplicity distribution within restricted phase space. The corresponding events have no particles within the restricted phase space,i.e., all the particles locate outside the restricted region, the so-called rapidity gap events. Even for the multiplicity distribution in the full phase space, the non-zero void probability is not difficult to interpret, as one does not record all the outgoing particles. The void probability is not only concerned with the proper normalization of the distribution, but also relates to underlying dynamics through the higher order correlations.

Consider the generating function $G(z)$ for multiplicity distribution $P(n)$ defined as

$$G(z) = \sum_{n=0}^{\infty} z^n \; P(n) \; . \tag{1}$$

The above relationship can be reversed to obtain

$$P(n) = \frac{1}{n!} \; \left(\frac{d}{dz} \right)^n G(z) \bigg|_{z=0} \; . \tag{2}$$

The proper normalization of $P(n)$ implies

$$1 = G(1) \; , \tag{3}$$

and the average multiplicity is given by

$$\langle n \rangle = \frac{d}{dz} G(z) \bigg|_{z=1} \; . \tag{4}$$

The void probability $P(0)$ is related to the generating function through

$$P(0) = G(0) \; , \tag{5}$$

and can be taken as another generating function for $P(n)$:

$$P(n) = (-1)^n \; \frac{\langle n \rangle^n}{n!} \left(\frac{\partial}{\partial \langle n \rangle} \right)^n P(0) \; , \tag{6}$$

where the differentiation is carried out with the correlation functions held fixed [1]. Besides serving as a generating function, the void probability is also related to the probability $P(n)$ with $n \neq 0$ through various kinds of moments. The void probability $P(0)$ can be written as an expansion either in factorial moments or in cumulants,

$$P(0) = \sum_{q=1}^{\infty} (-1)^q \; \frac{\langle n \rangle^q}{q!} \; F_q \; , \tag{7}$$

$$ln \; P(0) = \sum_{q=1}^{\infty} (-1)^q \; \frac{\langle n \rangle^q}{q!} \; K_q \; , \tag{8}$$

where F_q and K_q denote the normalized factorial moments and cumulants respectively. Therefore, the void probability provides a good discriminator among various theoretical models.

In data analysis, the low multiplicity region of multiple production is always contaminated with the processes from elastic scattering and diffraction dissociation. It is not easy to separate these mechanisms from one another by just observing the final hadron distribution. The accurate data for low multiplicity region including the void probability are scarce. In this paper we will study the low multiplicity region of multiple production with the widely used FRITIOF 7.02 model [2], which has had reasonable success in accounting for the experimental data. Here our objective is to study the predictions of the QCD-inspired FRITIOF model concerning the void probability of the multiple production, and to see if such predictions could be accommodated with the phenomenological clan model. The clan model is briefly reviewed in Sect. 2. In Sect. 3, we present the results from the numerical simulations of the FRITIOF model. The discussions and conclusion are presented in Sect. 4.

2 The clan model

In the clan model [3], the mechanism of multiple production is described as a two-step process. In the first step, the clans are produced randomly and distributed as a Poisson distribution,

$$P_1(n) = \frac{\lambda^n}{n!}\, e^{-\lambda} \ . \tag{9}$$

In the second step, each clan decays into final particles independently with a given distribution $P_2(n)$. As the clans are identified by the final particles, one requires that each clan produces at least one particle,i.e., $P_2(0) = 0$. The multiplicity distribution is then determined by the combination of $P_1(n)$ and $P_2(n)$. Specifically, the generating function of the resulted multiplicity distribution is the convolution of the generating functions for $P_1(n)$ and $P_2(n)$,

$$G(z) = G_1\left(G_2(z)\right) \ , \tag{10}$$

where $G_1(z)$ and $G_2(z)$ are the generating functions for $P_1(n)$ and $P_2(n)$ respectively.

When $P_2(n)$ is taken as the logarithmic distribution

$$P_2(n) \propto \frac{b^n}{n} \ , \tag{11}$$

the resulted multiplicity distribution is the negative binomial distribution [3]. When $P_2(n)$ is taken as the geometric distribution

$$P_2(n) \propto b^n \ , \tag{12}$$

the multiplicity distribution is the Ising distribution [4], which can be obtained analytically from the nearest-neighbor Ising model with lattice gas interpretation. There are only two free parameters in these models, λ in $P_1(n)$ and b in $P_2(n)$. In general, the distribution $P_2(n)$ can be determined from the multiplicity distribution $P(n)$ unambiguously. As the produced clan must decay,

$$P_2(0) = G_2(0) = 0 \ , \tag{13}$$

the void probability is given by

$$P(0) = G(0) = G_1(0) = P_1(0) = e^{-\lambda} \ . \tag{14}$$

The generating function is given by

$$G(z) = e^{\lambda[G_2(z)-1]} \ , \tag{15}$$

or equivalently

$$G_2(z) = 1 - \frac{1}{ln\ P(0)}\ ln\ G(z) \ . \tag{16}$$

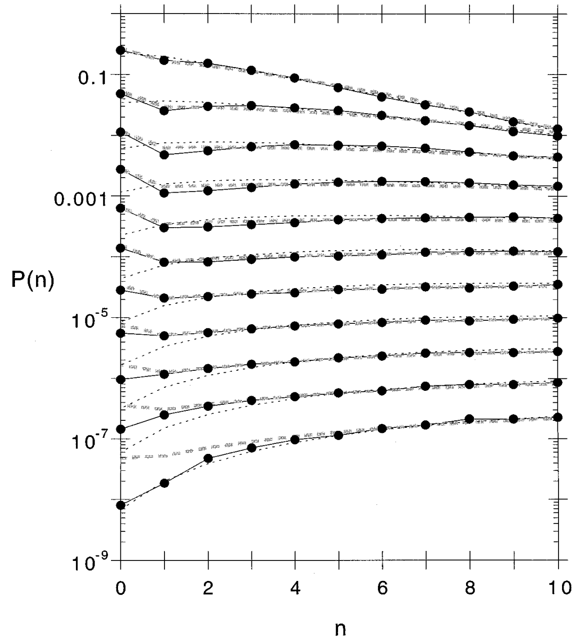

Fig. 1. Multiplicity distribution $P(n)$ v.s. n for charged $\pi^{\pm}$ in $\bar{p}p$ collisions at $\sqrt{s}$=540 GeV. The curves from top to bottom are for pseudorapidity intervals $\Delta\eta$=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and full phase space, respectively. The distribution for the smallest interval is shown in the ordinary scale, and each consecutive distribution is shifted down by a factor of $\sqrt{10}$. The data points connected by *solid lines* are the predictions of FRITIOF model. The *dotted lines* are fitted by the negative binomial distributions and the *grey bold-dashed lines* are fitted by the Ising distributions

For the first few n, the relationships between multiplicity distributions $P(n)$ and $P_2(n)$ are as follows

$$P_2(1) = -\frac{1}{ln\ P(0)}\left(\frac{P(1)}{P(0)}\right) \ , \tag{17}$$

$$P_2(2) = -\frac{1}{ln\ P(0)}\left(\frac{P(2)}{P(0)} - \frac{1}{2}\frac{P(1)^2}{P(0)^2}\right) \ , \tag{18}$$

$$P_2(3) = -\frac{1}{ln\ P(0)}\left(\frac{P(3)}{P(0)} - \frac{P(2)P(1)}{P(0)^2} + \frac{1}{3}\frac{P(1)^3}{P(0)^3}\right) \ , \tag{19}$$

$$P_2(4) = -\frac{1}{ln\ P(0)}\left(\frac{P(4)}{P(0)} - \frac{P(3)P(1)}{P(0)^2} - \frac{1}{2}\frac{P(2)^2}{P(0)^2} + \frac{P(2)P(1)^2}{P(0)^3} - \frac{1}{4}\frac{P(1)^4}{P(0)^4}\right) \ . \tag{20}$$

Without accurate estimation of the void probability $P(0)$, the above relationships are hard to evaluate.

3 The FRITIOF model

Since the accurate data for the void probability are scarce, we use the Monte Carlo program FRITIOF 7.02 [2] as an event generator to study the low multiplicity region in high-energy multiple productions. An ensemble of 5×10^4

minimum biased events is generated with default parameters for $\bar{p}p$ collision at CERN S$\bar{\rm p}$pS energy $\sqrt{s}$=540 GeV. As the final particles are mainly pions, we record only π^+, π^-, and π^0 exclusively. The resulted multiplicity distributions for three types of pions and any of their combinations are similar, which implies that the showing features are the results of production mechanism in general, instead of the specific dynamics like resonance. The multiplicity distributions in the low multiplicity region including the void probability for various pseudorapidity intervals $\Delta\eta$ are shown in Fig. 1 for charged pions. As shown in the figure, the values of $P(0)$ are not equivalent to those expected by smooth extrapolation from $P(n)$ at $n \neq 0$. A dip is observed at $P(1)$,i.e.,

$$P(0) > P(1) < P(2) \ . \tag{21}$$

The void probability is greatly enhanced, especially in the restricted pseudorapidity intervals. For the multiplicity distributions of the single species, π^-, π^+, or π^0, the enhancement of void probability occurs at the pseudorapidity intervals $2 \le \Delta\eta \le 8$. For the distributions of all pions or any of their combinations, the void probability enhances even for the smallest interval $\Delta\eta = 1$,i.e., the dip is observed for the intervals $\Delta\eta \le 8$. As these observed features are the same for all pions, it is more likely that they are originated from statistical effects in general rather than from specific effects like resonance.

The predictions from the clan models is also shown in the same figure for the negative binomial distributions [3] and the Ising distributions [4]. The two parameters in the clan models are fixed by the values of the average multiplicity $\langle n\rangle$ and the dispersion $D \equiv \sqrt{\langle n^2\rangle - \langle n\rangle^2}$ in the simulations. Since the best-fitting process is not performed, the shape of the distribution constitutes the predictions of the two models. For large multiplicity $n \ge 5$, these two models give almost the same result. Their difference is in the predictions of the low multiplicity region, especially the void probability. The negative binomial distribution describes the multiplicity distribution in the full phase space quite well, but fails to predict any dip at $P(1)$. The values of the void probability are underestimated for the restricted phase space. On the contrary, the Ising distribution overestimates the void probability in the full phase space, but the dip at $P(1)$ for the restricted phase space is fully reproduced. For the intervals $\Delta\eta \le 7$, the values of the void probability are also well described by the Ising distribution. In summary, the multiplicity distribution in the full phase space is well described by the negative binomial distribution, and the distribution in the restricted phase space by the Ising distribution.

To further study the energy dependence of the observed enhancement of the void probability, we generate the same number of events for various collision energies and then analyze the multiplicity distributions. The pseudorapidity ranges for the void probability enhancement increase with the increasing of energy. At $\sqrt{s}$=200 GeV, the enhancement occurs for the intervals $\Delta\eta \le 7$. At $\sqrt{s}$=540 GeV, the range of enhancement increases to $\Delta\eta \le 8$; at $\sqrt{s}$=900 and 1800 GeV, the range further increases to $\Delta\eta \le 9$ and $\Delta\eta \le 10$, respectively. The same features are observed as mentioned previously. We note that the inclusion of charged kaon $K^{\pm}$ does not change this observation.

4 Discussions and conclusion

We have shown that the accurate measurement of the void probability can be used to discriminate the underlying dynamics of multiple production. Within the simulations of the FRITIOF model, the resulted multiplicity distribution can not be well described by the negative binomial distribution. A systematic deviation is observed. Though the overall shape of the distribution can be fitted by adjusting the two free parameters, the low multiplicity region is underestimated, especially the void probability. We find that the distributions can be well described by Ising distribution and negative binomial distribution respectively for the very small phase space and the full phase space. In the clan model, this means that the decay distribution in the restricted region is geometric distribution and that in the unrestricted region is logarithmic. All the multiplicity distributions observed are in-between these two limits.

The geometric distribution occurs quite naturally in the parton shower model where the clans are identified as the bremsstrahlung gluon jets. This distribution corresponds to the simplest self-similar cascade process and is expected to be a good approximation for QCD gluon jets [5]. The generating function for the geometric distribution is given by

$$G_{geo}(z,\nu) = \frac{z}{\nu + z - \nu z} \ , \tag{22}$$

where the parameter ν provides a clear intuition as the average multiplicity of the decay distribution and is related to the parameter b of (12) through

$$\nu = \frac{1}{1-b} \ . \tag{23}$$

The logarithmic distribution can be taken as an average over geometric distributions and the corresponding generating function can be written as

$$G_{log}(z,\nu) = \frac{\displaystyle\int_1^{\nu} G_{geo}(z,\nu') \ \frac{d\nu'}{\nu'}}{\displaystyle\int_1^{\nu} \frac{d\nu'}{\nu'}} \ , \tag{24}$$

where the lower limit of integration is set to $\nu' = 1$ as the produced clans must decay, and the average is performed with weight $\frac{1}{\nu'}$. To our knowledge, there is no clear illustration why the weighting factor has to be $\frac{1}{\nu'}$. To accommodate the observed features of enhanced void probability, we have to chose different weighting factors for different pseudorapidity intervals. Phenomenologically

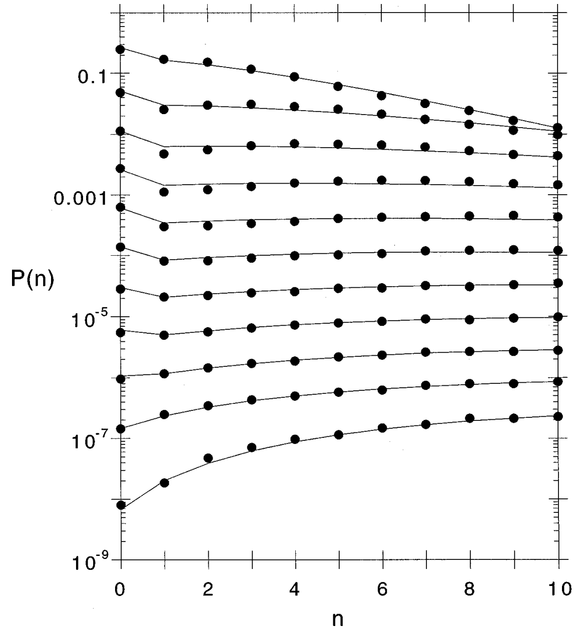

Fig. 2. Multiplicity distributions $P(n)$ v.s. n for charged $\pi^\pm$ in $\bar{p}p$ collisions at $\sqrt{s}$=540 GeV. The data points are the predictions of FRITIOF model, the same as in Fig. 1. The *solid lines* are the predictions of the clan model with parameter α defined in (26)

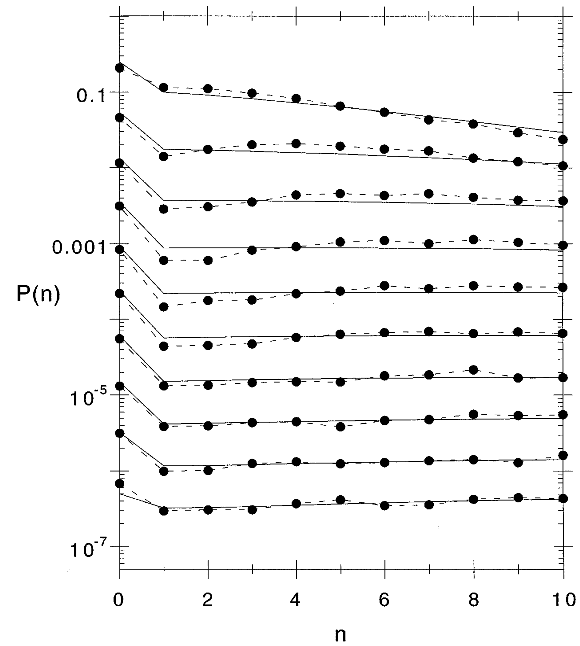

Fig. 3. The same as in Fig. 2 for charged $\pi^\pm$ at $\sqrt{s}$=10 TeV. The data points connected by *dashed lines* are from the FRITIOF model. The *solid lines* are the predictions of the clan model

a new parameter α is introduced and the generating function is written as

$$G_2(z,\nu) = \frac{\displaystyle\int_1^\nu G_{geo}(z,\nu')\ \nu'^\alpha d\nu'}{\displaystyle\int_1^\nu \nu'^\alpha d\nu'} , \qquad (25)$$

i.e., the weighting factor is parameterized to be ν'^α. The logarithmic and geometric distributions can be reproduced by $\alpha = -1$ and $\alpha \to \infty$ respectively. As α varies from -1 to ∞, the weight is shifted from $\nu' = 1$ to $\nu' = \nu$ continuously. The multiplicity distribution for the full phase space can be well described by $\alpha = -1$. For the restricted phase space, the distributions can be reproduced by the following parameterization

$$\alpha = 10 - \Delta\eta , \qquad (26)$$

where a simple linear dependence on the pseudorapidity interval $\Delta\eta$ is assumed. The results for the distribution at $\sqrt{s}$ = 540 GeV are shown in Fig. 2. We note that these are not the results of best fit, the two parameters in the clan model being simply fixed by the values of the average multiplicity $\langle n\rangle$ and the dispersion $D \equiv \sqrt{\langle n^2\rangle - \langle n\rangle^2}$ in the simulations as before. The enhancement of the void probability can be well described by this simple parameterization. The predictions at $\sqrt{s}$=10 TeV is shown in Fig. 3.

To conclude, we point out that the abundant experimental evidence on the occurrence of negative binomial distribution in high energy collisions examines mainly the distribution in the large multiplicity region. The low multiplicity region including the void probability is often overlooked. We strongly suggest a re-examination of the multiplicity distribution focused on the void probability. We expect the discontinuous enhancement of the void probability shown in the FRITIOF model should also be confirmed in the experimental data. The shift of the weighting factors in averaging the geometric distributions could also provide deeper understanding to the detailed dynamics of the multiple productions.

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